

Scattering optimization of photonic cluster: from minimal to maximal reflectivity

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Abstract

The optimization of the light scattered by photonic cluster made of small particles is studied with the help of the local perturbation method and special optimization algorithm. It was shown that photonic cluster can be optimized in a such a way that its reflectivity will be increased or decreased by several orders of magnitude for selected wavelength and direction.

1 Introduction

Light is known as fastest carrier of energy and information and this property makes light indispensable for communications, warfare and fundamental research. While light can be relatively easy created and guided, its manipulation is somewhat difficult. Manipulation of the light requires dynamical control of the refractive index of the host medium. While the materials for active control of the light are still under development, the theoretical studies are already started. Recently, the light manipulation was investigated in work [1] where the weak scattering by the finite object was studied by using the first Born approximation. In practice, the light scattering by photonic

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cluster may be not weak and it should be studied with other methods. The local perturbation method (LPM) is suitable tool for such analysis. The LPM correctly describes scattering by particles with arbitrary large refractive index which are small compared to the incident wavelength and the scattering can be strong (see for example work [2]-[4] and references therein). In works [5]-[7] the LPM was used to study the wave propagation in the photonic cluster.

In this paper we optimize the light scattering from the cluster made of small particles by using the LPM and special optimization technic. By using our method we modify the cluster in such a way that the scattering from the cluster is significantly minimized or maximized at one point. We present several examples which will show that the scattering by the cluster can be increased and decreased by several orders of magnitude for selected wavelength and direction.

2 The formalism

The formalism we use is presented in many works (see for example [2]-[4]) and we only briefly present it here for convenience and consistency. Consider the photonic cluster made of particles which characteristic sizes are small compared to the incident wavelength λ_0 . The electric field \mathbf{E} propagating in the host medium filled with N particles is described by the following equation [4]

$$(\Delta - \nabla \otimes \nabla + k_0^2) \mathbf{E}(\mathbf{r}) + \frac{\omega^2}{c^2} \sum_{n=0}^{N-1} (\varepsilon_{sc,n} - \varepsilon_0) f_n(\mathbf{r} - \mathbf{r}_n) \mathbf{E}(\mathbf{r}_n) = \mathbf{S}(\mathbf{r}), \quad (1)$$

where

$$k_0 = |\mathbf{k}_0| = \frac{2\pi}{\lambda_0} = \frac{\omega}{c} \sqrt{\varepsilon_0}, \quad f_n(\mathbf{r} - \mathbf{r}_n) = \begin{cases} 1, & \text{inside particle} \\ 0, & \text{outside particle} \end{cases}.$$

Here Δ is Laplacian and ∇ nabla operators, \otimes defines tensor product, k_0 is a wave number in the host medium (ω is the angular frequency and c is

the light velocity in vacuum), $\varepsilon_{sc,n}$ and ε_0 are the permittivity of the n -th particle and the medium respectively, f_n is the function describing the shape of the n -th scatterer, and \mathbf{S} is the field source. The characteristic size of n -th scatterer we denote as L_n . Note, that the equation (1) is an approximate one and this is true only when the small scatterers ($k_0 L_n \ll 1$) are considered. This equation is easily solvable with respect to the fields $E(\mathbf{r}_n)$ when the positions \mathbf{r}_n are known (it will invoke solution of $3N$ linear equations for each frequency ω). The solution of the equation (1) can be written in the following form

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) + \mathbf{E}_{sc}(\mathbf{r}), \quad (2)$$

where

$$\mathbf{E}_{sc}(\mathbf{r}) = \frac{\omega^2}{c^2} \left(\hat{I} + \frac{\nabla \otimes \nabla}{k_0^2} \right) \sum_{n=0}^{N-1} \mathbf{E}(\mathbf{r}_n) (\varepsilon_{sc,n} - \varepsilon_0) \Phi_n(\mathbf{r}) \quad (3)$$

and

$$\Phi_n(\mathbf{r}) = \int_{-\infty}^{\infty} \frac{\tilde{f}_n(\mathbf{q}) e^{i\mathbf{q} \cdot (\mathbf{r} - \mathbf{r}_n)}}{(q^2 - k_0^2)} d\mathbf{q}, \quad \tilde{f}_n(\mathbf{q}) = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} f_n(\mathbf{r}) e^{-i\mathbf{q} \cdot \mathbf{r}} d\mathbf{r}. \quad (4)$$

Here \hat{I} is the 3×3 unitary tensor in polarization space and \mathbf{r}_n is the radius vector of the n -th particle. The field $\mathbf{E}(\mathbf{r}_n)$ is the field inside the n -th particle, \tilde{f}_n is the Fourier transform of the function f_n , and \cdot defines scalar product. The incident field \mathbf{E}_0 is created by the source \mathbf{S} in the host medium and it is not important for our consideration.

The formula (2) is rather general one and it describes the field in the medium with photonic cluster of arbitrary form made of small particles of arbitrary form.

The fields $\mathbf{E}(\mathbf{r}_n)$ should be found by solving the system of $3N$ linear equations obtained by substituting $\mathbf{r} = \mathbf{r}_n$ into Eq. (2). The formula for the scattered field (3) can be simplified even further when the distance between the observer and an n -th scatterer (R_n) is large, i.e. when $R_n \gg L_n$. In this case the integral Φ_n can be calculated approximately. We note also that

the integral Φ_n can be calculated exactly at least for the spherical particles. When $R_n \gg L_n$ integration in (3) gives

$$\mathbf{E}_{sc}(\mathbf{r}) = \frac{\omega^2}{4\pi c^2} \left(\hat{I} + \frac{\nabla \otimes \nabla}{k_0^2} \right) \sum_{n=0}^{N-1} \mathbf{E}(\mathbf{r}_n) (\varepsilon_{sc,n} - \varepsilon_0) \frac{e^{ik_0 R_n}}{R_n} V_n, \quad (5)$$

where

$$R_n = |\mathbf{r} - \mathbf{r}_n| \gg L_n. \quad (6)$$

Here R_n is the distance between the observation point and n -th scatterer, V_n is the volume of the n -th scatterer, and $|\dots|$ brackets denote absolute value.

In practice, the distance between the cluster and the observer is much more larger than the size of the cluster and the inequality $|\mathbf{r}| \gg \max(|\mathbf{r}_n|)$ is fulfilled. In this case the field (5) can be simplified and rewritten in the form

$$\mathbf{E}_{sc}(\mathbf{r}) = \frac{\omega^2 e^{ik_0 r}}{4\pi c^2 r} \left(\hat{I} - \mathbf{l} \otimes \mathbf{l} \right) \sum_{n=0}^{N-1} \mathbf{E}(\mathbf{r}_n) V_n (\varepsilon_{sc,n} - \varepsilon_0) e^{-ik_0 \mathbf{l} \cdot \mathbf{r}_n}, \quad (7)$$

where

$$\mathbf{l} = \mathbf{r}/r, \quad r = |\mathbf{r}| \gg \max(|\mathbf{r}_n|).$$

3 Optimization of the cluster reflectivity

The idea of our approach is the following. We assume that scatterers in the cluster can be repositioned in a such a way that the field scattered by the cluster will be tuned to required value at the observation point.

At first, we calculate the intensity of the scattered field from the given (not modified) cluster for selected wavelength λ_{opt} and observation point \mathbf{r} . After this we select one particle in the cluster and reposition it a bit in x , y , and z directions (usually for tenth of the particle size) recalculating the scattered field in \mathbf{r} each time we move the particle. From the array of the calculated intensities we select the value nearest to the required one and we place the particle at the point corresponding to this intensity value. We perform this procedure for all particles in the cluster and repeat it several

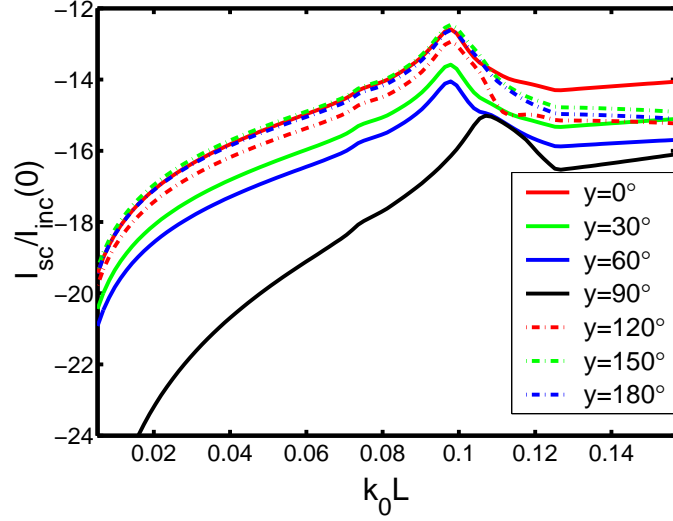


Figure 1: The logarithm of the normalized intensity of the scattered field $I_{sc}(\mathbf{r})/I_{inc}(0)$ versus $k_0 L$ for the not optimized *Au* cluster. The particles in the cluster are arranged into cubic lattice. The permittivity of the host medium is $\varepsilon_0 = 1$, the characteristic size of the particles is $L = 10$ nm, the period of the cluster is $d = 3.3L$, and the total number of the particles is $N = 123$. The source is positioned at $\mathbf{r}_s = \{x_s, 0, 0\}$ and the observation points \mathbf{r} are located at the circle plane with radius x_s . The angle θ is the angle between vector \mathbf{r} and \mathbf{r}_s in xy plane.

times. After several iterations, the particles in the cluster will be rearranged in such a way that the intensity of the scattered light will correspond the required value at the selected observation point \mathbf{r} and for the wavelength λ_{opt} .

For example, super reflective cluster can be created when the intensity of the scattered field $I_{sc} = |\mathbf{E}_{sc}(\mathbf{r})|^2$ will be maximized at some point and minimized at all other points (directions). Another important example closely related to optical cloaking is the cluster with significantly reduced reflectivity for selected wavelength at chosen direction. When the reflectivity of the cluster is lower than the sensitivity level of the receiver, this cluster is actually invisible for observer.

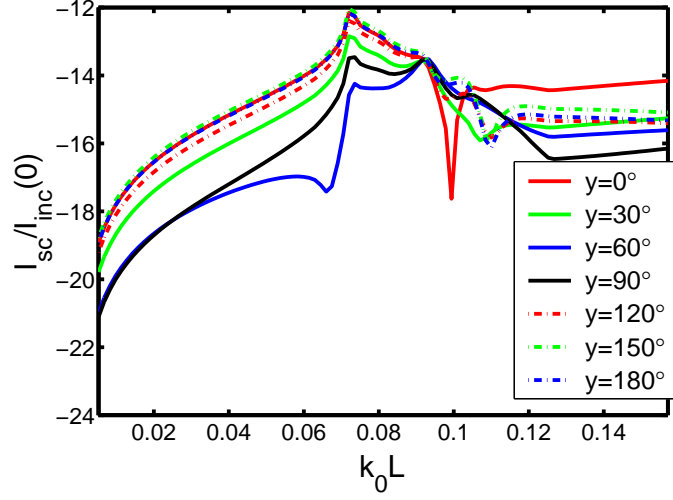


Figure 2: The logarithm of the normalized intensity of the scattered field $I_{sc}(\mathbf{r})/I_{inc}(0)$ versus $k_0 L$ for the optimized spherical cluster made of *Au* cubes. The permittivity of the host medium is $\varepsilon_0 = 1$, the characteristic size of the cubes is $L = 10$ nm and the total number of the particles is $N = 123$. The source is positioned at $\mathbf{r}_s = \{x_s, 0, 0\}$ and the observation points \mathbf{r} are located at the circle with radius x_s . The angle θ is the angle between vector \mathbf{r} and \mathbf{r}_s in xy plane. The cluster reflectivity was minimized for $k_0 L = 0.1$ at $\mathbf{r} = \mathbf{r}_s$. The red line shows extremely low deep in cluster reflectivity after the optimization.

Moreover, when the intensity of the field scattered by the cluster is changed in some direction, this can be used in directional optical switch. This kind of device can be used for light houses, for example. When the spectrum of the field scattered by the cluster is modified in time when the cluster can be used as a filter.

Below we present several examples demonstrating the possibility to optimize the cluster reflectivity in chosen direction for selected wavelength.

We note that similar approach was successfully used in work [8] where the phase of incident field was tuned to maximize the field scattered from the complex object (eggshell and TiO powder) in chosen direction.

3.1 Examples

Consider the spherical photonic cluster consisting of *Au* cubes. The characteristic size of the cubes is L and they are organized into simple cubic lattice with period d . The center of the cluster is positioned at the beginning of the coordinates. The incident field is generated by the point source positioned at the point $\mathbf{r}_s = \{x_s, 0, 0\}$ and it is linearly polarized in y direction. The scattering by the cluster will be optimized (to minimum or maximum) for the chosen wavelength λ_{opt} for the observer positioned at the point \mathbf{r} . We assume that the permittivity of the scatterers is the same as the permittivity of the bulk *Au* and the actual values of permittivity were taken from [9]. Note that the source and the observer are far from the cluster such that $k_0 |\mathbf{r}_s| \gg 1$ and $k_0 |\mathbf{r}| \gg 1$.

The examples of the optimization are presented on Figs. 1-5. The figures show the normalized intensity of the light scattered by the clusters versus $k_0 L$. The Fig. 1 shows the light intensity scattered from the not optimized cluster. Figs. 2 and 3 show the results for the clusters which scattering was optimized at the point $\mathbf{r} = \mathbf{r}_s$ and the Figs. 4 and 5 show the results for the clusters which scattering was optimized at the point $\mathbf{r} = \{0, x_s, 0\}$.

The positions of the particles in the clusters were changed in order to maximize or minimize the scattering for the wavelength $\lambda_{opt} = 633$ nm ($k_0 L = 0.1$) in chosen directions. The results are presented for different angles of observation θ in xy plane. The Fig. 2 shows normalized (with respect to the intensity of the incident field $I_{inc}(0)$) intensity of the light $I_{sc}(\mathbf{r}) = |\mathbf{E}_{sc}(\mathbf{r})|^2$ scattered by the optimized photonic cluster. The cluster was optimized to minimal intensity of the scattered field in the direction of the source ($\theta = 0$). Comparing Fig. 1 and Fig. 2 one can see huge deep (about five orders of magnitude) in the intensity of the scattered field for the selected wavelength λ_{opt} and the selected direction $\theta = 0$. The Fig. 2 shows also that unintendently, the optimization significantly increased intensity of the scattered field for other wavelengths (near $k_0 L = 0.07$) in almost all directions.

The Fig. 3 shows normalized intensity of the light scattered by the cluster optimized to maximal intensity of the scattered field in the direction $\theta = 0$. Comparing Fig. 1 and Fig. 3 one can see relatively large increase (about two

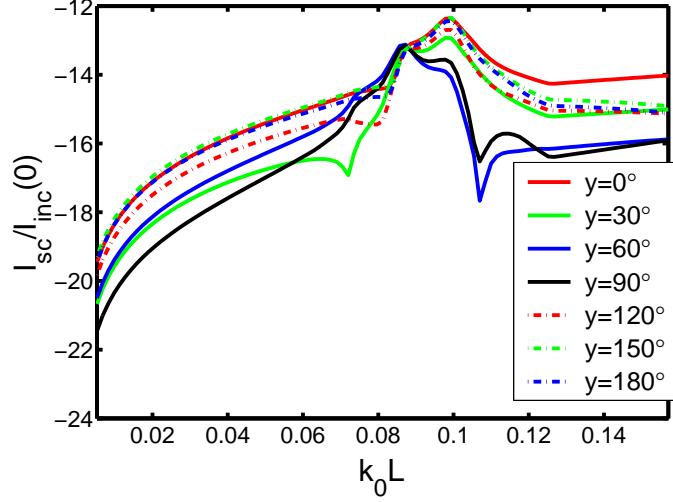


Figure 3: The logarithm of the normalized intensity of the scattered field $I_{sc}(\mathbf{r})/I_{inc}(0)$ versus $k_0 L$ for the optimized spherical cluster made of *Au* cubes. The permittivity of the host medium is $\varepsilon_0 = 1$, the characteristic size of the cubes is $L = 10$ nm and the total number of the particles is $N = 123$. The source is positioned at $\mathbf{r}_s = \{x_s, 0, 0\}$ and the observation points \mathbf{r} are located at the circle with radius x_s . The angle θ is the angle between vector \mathbf{r} and \mathbf{r}_s in xy plane. The cluster reflectivity was maximized for $k_0 L = 0.1$ at $\mathbf{r} = \mathbf{r}_s$. The red line shows extremely high peak in cluster reflectivity after the optimization.

times) in the intensity of the scattered field for the selected wavelength λ_{opt} and the selected direction $\theta = 0$. The Fig. 3 shows also that the optimization significantly increased intensity of the scattered field for $k_0 L = 0.09$ and significantly decreased the intensity for $k_0 L = 0.11$ in directions with $\theta = \pi/2$ and $\theta = \pi/3$ respectively.

The Fig. 4 shows normalized intensity of the light scattered by the cluster optimized to minimal intensity of the scattered field in the direction perpendicular to the source-cluster direction ($\theta = \pi/2$). Comparing Fig. 1 and Fig. 4 one can see significant decrease (about two orders of magnitude) in the intensity of the scattered field for the selected wavelength λ_{opt} in the selected

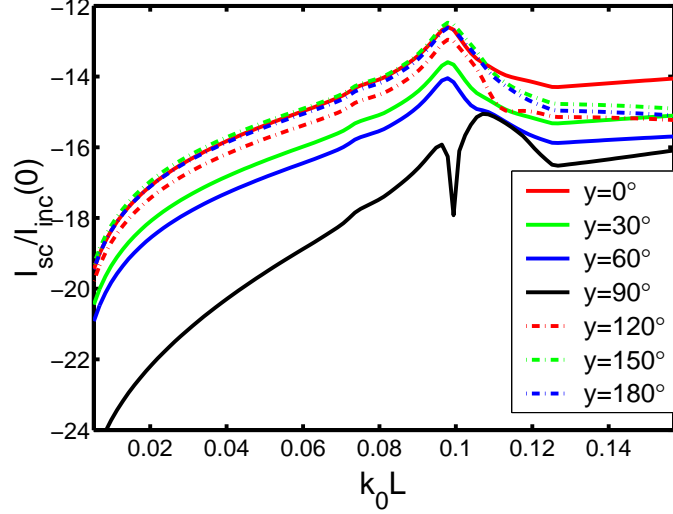


Figure 4: The logarithm of the normalized intensity of the scattered field $I_{sc}(\mathbf{r})/I_{inc}(0)$ versus k_0L for the optimized spherical cluster made of Au cubes. The permittivity of the host medium is $\varepsilon_0 = 1$, the characteristic size of the cubes is $L = 10$ nm and the total number of the particles is $N = 123$. The source is positioned at $\mathbf{r}_s = \{x_s, 0, 0\}$ and the observation points \mathbf{r} are located at the circle with radius x_s . The angle θ is the angle between vector \mathbf{r} and \mathbf{r}_s in xy plane. The cluster reflectivity was minimized for $k_0L = 0.1$ at $\mathbf{r} = \{0, x_s, 0\}$. The red line shows extremely low deep in cluster reflectivity after the optimization.

direction $\theta = \pi/2$. In distinction to previous examples, the intensity of the scattered field in other directions is almost not affected in this case.

The Fig. 5 shows normalized intensity of the light scattered by the cluster optimized to maximal intensity of the scattered field in the direction $\theta = \pi/2$. Comparing Fig. 1 and Fig. 5 one can see huge increase (about three orders of magnitude) in the intensity of the scattered field for the selected wavelength λ_{opt} in the selected direction $\theta = \pi/2$. We note that the intensity of the scattered field increased also in direction $\theta = \pi/3$ (for λ_{opt}) and for all other directions the intensity peak shifted to $k_0L = 0.09$.

Examination of the Figs. 1-5 reveals several important things. The first

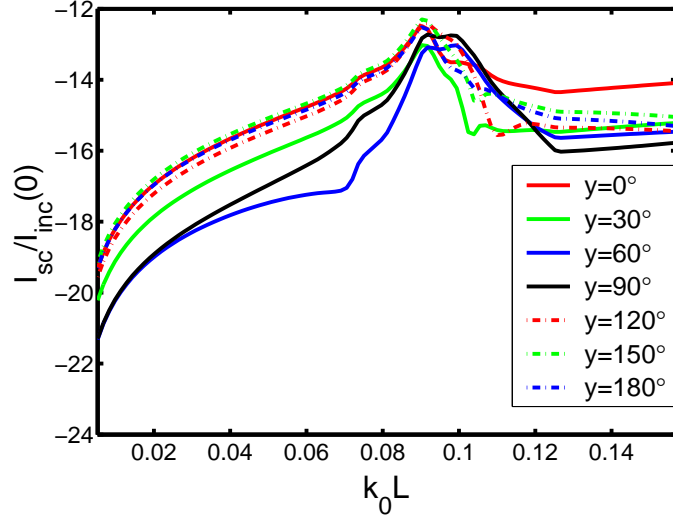


Figure 5: The logarithm of the normalized intensity of the scattered field $I_{sc}(\mathbf{r})/I_{inc}(0)$ versus $k_0 L$ for the optimized spherical cluster made of *Au* cubes. The permittivity of the host medium is $\varepsilon_0 = 1$, the characteristic size of the cubes is $L = 10$ nm and the total number of the particles is $N = 123$. The source is positioned at $\mathbf{r}_s = \{x_s, 0, 0\}$ and the observation points \mathbf{r} are located at the circle with radius x_s . The angle θ is the angle between vector \mathbf{r} and \mathbf{r}_s in xy plane. The cluster reflectivity was maximized for $k_0 L = 0.1$ at $\mathbf{r} = \{0, x_s, 0\}$. The black line shows extremely high peak in cluster reflectivity after the optimization.

one is that the optimization works because the intensity of the light scattered by the optimized cluster has very clear minima or maxima for the chosen wavelengths and positions. The second one is that the intensity of the scattered field has deep or peak not only for designed wavelength but for other wavelengths and directions. It should be emphasized that the scattering was optimized for one direction and for one wavelength only and that is why the optimization is effective for narrow spectrum of wavelengths and scattering directions.

4 Conclusions

The reflectivity of the spherical photonic cluster made of small particles was optimized to maximal and minimal values by using the local perturbation method and the special optimization algorithm. The possibility to design the photonic cluster with required scattering characteristics was demonstrated for selected wavelength and direction.

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